# PHYS 301
# Electricity and Magnetism

Dr. Gregory W. Clark

Fall 2019

---

## Today!

- Quiz!!
- The Divergence
- The Curl
- Intro to spherical coordinates

## **Differential Calculus:** Del on vectors?

- The **del operator** is a **differential operator** that "*acts on*," rather than "multiplies" the function to its right.
- Acting on vectors, we have two optiosn of interest:

$\vec{\nabla} \cdot \vec{V}$ **The Divergence**

$\vec{\nabla} \times \vec{V}$ **The Curl**

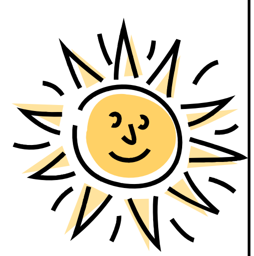
<u>**NOTE:**</u> *like the gradient, these are **COORDINATE SYSTEM DEPENDENT**!!*

## **Differential Calculus:** The Divergence

- Gives feel for how much the field is *spreading out* or DIVERGING from the point in question!
- Often associate with *sources* or *sinks* of the field.
- Sort of a "*slope of the components*."
- Results in a scalar function!

$$\vec{\nabla} \cdot \vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

**The Divergence in Cartesian coords.**



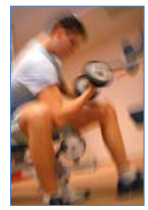
*[**COORDINATE SYSTEM DEPENDENT!!**]*

# **Differential Calculus:** The Curl

- Gives feel for how much the vector field is *rotating* or CURLING about the point in question!
- Results in a vector function.

$$\vec{\nabla}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z}\\ B_x & B_y & B_z\end{vmatrix}$$

or



$$\vec{\nabla}\times\vec{B}=\hat{i}\left(\frac{\partial B_z}{\partial y}-\frac{\partial B_y}{\partial z}\right)+\hat{j}\left(\frac{\partial B_x}{\partial z}-\frac{\partial B_z}{\partial x}\right)+\hat{k}\left(\frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}\right)$$

**The Curl in Cartesian coords.**

*[COORDINATE SYSTEM DEPENDENT!!]*

# **Differential Calculus:**

- Things change in other coordinate systems:

Cylindrical

$$\nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}}+\frac{1}{s}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}+\frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

$$\nabla\cdot\mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s)+\frac{1}{s}\frac{\partial v_\phi}{\partial \phi}+\frac{\partial v_z}{\partial z}$$

$$\nabla\times\mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial \phi}-\frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}}+\left[\frac{\partial v_s}{\partial z}-\frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}}+\frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi)-\frac{\partial v_s}{\partial \phi}\right]\hat{\mathbf{z}}$$

Spherical

$$\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}}+\frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}}+\frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$$

$$\nabla\cdot\mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, v_\theta)+\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

$$\nabla\times\mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta\, v_\phi)-\frac{\partial v_\theta}{\partial \phi}\right]\hat{\mathbf{r}}+\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \phi}-\frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}}+\frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta)-\frac{\partial v_r}{\partial \theta}\right]\hat{\boldsymbol{\phi}}$$

# Differential Calculus: MORE!

- Product Rules:
  - ∃ two for gradients
  - ∃ two for divergences
  - ∃ two for curls

See inside front cover of text!

- Second derivatives: ∃ only one of real interest to us:
  - The **LaPlacian:** the divergence of a gradient

$$\vec{\nabla}^2 f \equiv \vec{\nabla}\cdot\left(\vec{\nabla} f\right) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

## Spherical Coordinates

$r$ = radial distance

$\theta$ = polar angle

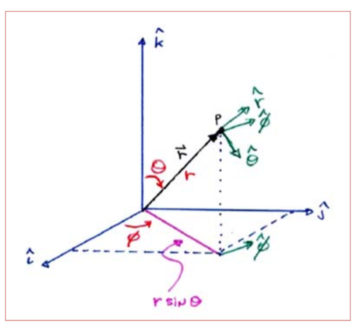
$\phi$ = azimuthal angle

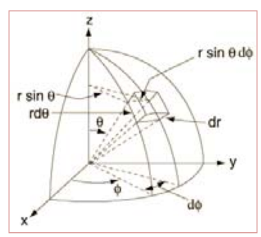
Unit vectors: $\hat{r}, \hat{\theta}, \hat{\phi}$

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$




$r = \left[x^2 + y^2 + z^2\right]^{1/2}$

$\theta = \tan^{-1}\left[\sqrt{x^2 + y^2}/z\right]$

$\phi = \tan^{-1}\left[y/x\right]$

$\hat{r} = \sin\theta\cos\phi\,\hat{\imath} + \sin\theta\sin\phi\,\hat{\jmath} + \cos\theta\,\hat{k}$

$\hat{\theta} = \cos\theta\cos\phi\,\hat{\imath} + \cos\theta\sin\phi\,\hat{\jmath} - \sin\theta\,\hat{k}$

$\hat{\phi} = -\sin\phi\,\hat{\imath} + \cos\phi\,\hat{\jmath}$

$\hat{\imath} = \sin\theta\cos\phi\,\hat{r} + \cos\theta\cos\phi\,\hat{\theta} - \sin\phi\,\hat{\phi}$

$\hat{\jmath} = \sin\theta\sin\phi\,\hat{r} + \cos\theta\sin\phi\,\hat{\theta} + \cos\phi\,\hat{\phi}$

$\hat{k} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$

$d\vec{l} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$

$d\tau = dV = r^2\sin\theta\,dr\,d\theta\,d\phi$

$d\vec{A}$ depends on surface!

**DIV, GRAD, and CURL are no longer so simple!!**

# Integral Calculus

- What is this familiar friend called?

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$\text{where} \quad f(z) = \frac{dF(z)}{dz}$$

and *f(z)* is continuous on *[a, b]*

**The Fundamental Theorem of Calculus**